Maine-Québec
Number Theory Conference

University of Maine
September 30$^{\text{th}}$ and October 1$^{\text{st}}$, 2023

Organizers

Jack Buttcane
Brandon Hanson
Andrew Knightly
Gil Moss

Saturday				
	DPC 105	DPC 107	DPC 115	DPC 117
8:55-9:00 DPC 100	Opening Remarks **Emily Haddad** Dean, College of Liberal Arts & Sciences			
9:00-9:50 DPC 100	**David Helm** (Imperial College London) *Finiteness for Hecke algebras of p-adic reductive groups*			
9:50-10:00	Break			
10:00-10:20	**Eran Assaf** (Dartmouth College) *Geometric Invariants of Hilbert Modular Surfaces*	**Félix Baril Boudreau** (University of Lethbridge) *Arithmetic Rank Bounds for Abelian Varieties*	**Alessandro Fazzari** (Université de Montréal) *Joint moments of the Riemann zeta function and its logarithmic derivative*	**Jon Grantham** (IDA/CCS) *No new Goormaghtigh primes up to* 10^{500}
10:20-10:30	Break			
10:30-10:50	**Romain Branchereau** (McGill University) *An upper bound on the denominator of Eisenstein classes in Bianchi manifolds*	**Garen Chiloyan** (none) *A classification of 2-adic Galois images of isogeny-torsion graphs*	**Matilde Lalin** (Universite de Montreal) *The distribution of values of cubic L-functions at $s = 1$*	**Paul Kinlaw** (Dickinson/UMaine Fort Kent) *Sums Related to the Fibonacci Sequence*
10:50-11:20	Coffee			
11:20-11:40	**Claire Frechette** (Boston College) *Metaplectic Ice: Lattice Models for General Metaplectic Whittaker Functions*	**Michael Chou** (Providence College) *Rank growth in dihedral extensions*	**Jakob Streipel** (University of Maine) *Zero-density estimate for Hecke–Maass L-functions*	**Tristan Phillips** (Dartmouth College) *Average analytic ranks of elliptic curves over number fields*
11:40-11:50	Break			
11:50-12:10	**Robin Zhang** (Mass. Inst. Tech.) *Harris-Venkatesh plus Stark*	**John Cullinan** (Bard College) *Probability of non-iso. gp. structures of isogenous elliptic curves in fin. fld ext's*	**Hugo Chapdelaine** (Laval University) *Computation of Stark numbers in abelian extensions of real quad. fields using Colmez*	**Michael Mossinghoff** (CCR Princeton) *Ideal solutions in the Prouhet-Tarry-Escott problem*
12:10-1:50	Lunch			
1:50-2:40 DPC 100	**Eric Urban** (Columbia) *On Euler systems for p-ordinary Galois representations*			
2:40-2:50	Break			
2:50-3:05 (students)	**Jacksyn Bakeberg** (Boston University) *Mod-ℓ gamma factors and a converse theorem for finite general linear groups*	**Asimina Hamakiotes** (University of Connecticut) *Elliptic curves with complex multiplication and abelian division fields*	**Vittoria Cristante** (Tufts University) *An Improved Lower Bound for $GL_2(\mathbb{F}_\ell)$ Number Fields*	**Steven Creech** (Brown University) *Explicit Zero Free Regions of Automorphic L-Functioons*
3:05-3:35	Coffee			
3:35-3:50 (students)	**Firdavs Rakhmonov** (University of Rochester) *Distribution of similar configurations in subsets of F_q^d*	**Batterman, et. al** *Biases in second moments of elliptic curves*	**Aditya Ghosh** (Columbia University) *Counting prime ideals of a given degree in number fields*	**Devadatta Ganesh Hegde** (University of Minnesota) *Poles of Degenerate Eisenstein Series and the Spectral Decomp of Aut Forms on $GL(n)$*
3:50-4:00	Break			
4:00-4:20	**Eyal Goren** (McGill University) *Foliations on Shimura varieties*	**Antonio Lei** (University of Ottawa) *On ordinary isogeny graphs with level structure*	**Erman Isik** (University of Ottawa) *Modular approach to the Diophantine equation $x^p + y^p = z^3$ over some # flds*	**Tung Nguyen** (Western University) *On the arithmetic of Fekete polynomials of principal Dirichlet characters*
4:20-4:30	Break			
4:30-4:50	**Mathilde Gerbelli-Gauthier** (McGill) *Fourier Interpolation and the Weil Representation*	**Adam Logan** (Gov. of Canada and ICERM) *Surfaces with maximal Picard number*		**Arseniy Sheydvasser** (Bates College) *Logical Rigidity of Arithmetic Groups*

Sunday				
	DPC 105	DPC 107	DPC 115	DPC 117
9:00-9:15 (students)	**Peikai Qi** (Michigan State University) *Iwasawa lambda Invariant and Massey product*	**Chentouf, Miller** *Patterns of primes in joint Sato–Tate distributions.*	**Siva Nair** (Universite de Montreal) *The Mahler measure of a family of polynomials*	**Marie-Anne Bourgie** (Université Laval) *Computing the lattice points of a convex rational polytope using Brion's formula*
9:15-9:25	Break			
9:25-9:40 (students)	**Martí Roset Julià** (McGill) *Dihedral long root local A-packets of G_2 via theta correspondence.*	**Batterman, et. al** *Modeling the vanishing of L-functions at the central point*	**Subham Roy** (Université de Montréal) *Areal Mahler measure and some examples*	**Eric Moss** (Boston College) *Computing Bianchi-Maass Forms*
9:40-10:10	Coffee			
10:10-10:25 (students)	**Benjamin York** (University of Connecticut) *On the adelic image of Galois representations attached to elliptic curves with cplx mult.*	**Ester Sgallová** (Michigan State University) *Prime sequences in discretely ordered principal ideal subring of $Q[x]$*	**Jae Hyung Sim** (Boston University) *Dedekind-Rademacher Cocycle and Explicit Class Field Theory*	
10:25-10:35	Break			
10:35-10:55	**Giovanni Rosso** (Concordia Univeristy) *Overconv. Eichler–Shimura morphism for families of Siegel modular forms*	**Arvind Suresh** (University of Arizona) *Genus g curves with rank $8g$ over real cyclotomic fields*	**David Lowry-Duda** (ICERM & Brown University) *Modular Murmurations*	
10:55-11:05	Break			
11:05-11:25	**Shiang Tang** (Dartmouth College) *Lifting G-valued Galois representations when $\ell \neq p$*	**Jonathan Love** (McGill) *On isospectral quaternion orders*	**Robert Lemke Oliver** (Tufts University) *Uniform exponent bounds on the number of primitive extensions of number fields*	
11:25-11:35	Break			
11:35-12:25 DPC 100	**Sarah Peluse** (University of Michigan) *Arithmetic patterns in dense sets*			

Abstracts

Eran Assaf, Dartmouth College
Geometric Invariants of Hilbert Modular Surfaces
Hilbert modular surfaces are 2-dimensional analogues of modular curves, in that they parametrize abelian surfaces with endomorphism and level structure. Similar to how curves are stratified by genus, surfaces are organized by their numerical invariants; the Enriques-Kodaira classification organizes smooth surfaces by Kodaira dimension, Hodge numbers, and Chern numbers. In this talk, I will explain how to compute these invariants for a Hilbert modular surface (with *various level structures*, thereby extending results of van der Geer). This is joint work with A. Babei, B. Breen, E. Costa, J. Duque-Rosero, A. Horawa, J. Kieffer, A. Kulkarni, G. Molnar, S. Schiavone and J. Voight. Our implementation can be found at https://github.com/edgarcosta/hilbertmodularforms.

Jacksyn Bakeberg, Boston University (student)
Mod-ℓ gamma factors and a converse theorem for finite general linear groups
Gamma factors are invariants attached to complex representations of groups such as $GL_n(F)$ where F is a local field or a finite field, and converse theorems assert that these gamma factors are complete invariants. In this talk, I will discuss recent work which defines gamma factors for representations in the finite field case, not with complex coefficients but instead with coefficients in a field of characteristic $\ell > 0$, and shows that they satisfy a converse theorem. This is joint with M. Gerbelli-Gauthier, H. Goodson, A. Iyengar, G. Moss, and R. Zhang.

Félix Baril Boudreau, University of Lethbridge
Arithmetic Rank Bounds for Abelian Varieties
In this talk, we examine the problem of bounding the rank of Abelian varieties defined over function fields. Let K be a function field with perfect constant field k of arbitrary characteristic. We give upper bounds, depending on K, on the rank of the Mordell-Weil group over K of any Abelian variety which has trivial K/k-trace. Our result generalizes in various ways a previous theorem by Jean Gillibert (Université de Toulouse) and Aaron Levin (Michigan State University) on elliptic curves over functions fields of characteristic different from 2 and 3 and is moreover stated under weaker assumptions. We also explore some consequences of our result. This is a joint work with Jean Gillibert and Aaron Levin.

Marie-Anne Bourgie, Université Laval (student)
Computing the lattice points of a convex rational polytope using Brion's formula
Let E be a vector space, M a lattice in E and P a rational convex polytope. Let V be the set of P's vertices. Brion proved that by adding the generating function of every vertex cone of P, we get a finite sum and the number of terms of this equation gives us the number of lattice points contained in P. In this talk we will discuss, from an intuitive point of view, how we can get to Brion's formula. We will also explain how we can get to Lawrence–Varchenko's formula starting from Brion's formula.

Romain Branchereau, McGill University
An upper bound on the denominator of Eisenstein classes in Bianchi manifolds
A general conjecture of Harder relates the denominator of the Eisenstein cohomology of certain locally symmetric spaces to special values of L-functions. We consider the locally symmetric space $\mathrm{SL}_2(\mathcal{O})\backslash\mathbb{H}^3$, where $\mathcal{O}$ is the ring of integers of an imaginary quadratic field. In that case, Berger proves a lower bound on the denominator of the Eisenstein cohomology in certain cases. In this talk, we will explain how results of Ito and Sczech can be used to prove an upper bound on the denominator in terms of a special value of a Hecke L-function. When the class number of K is one, we combine this result with Berger's result to obtain the exact denominator.

Hugo Chapdelaine, Laval University
Computation of Stark numbers in abelian extensions of real quadratic fields using the Colmez trick
Let K be a fixed totally real field. The rank one Stark conjecture predicts that certain partial zeta functions of K (twisted by a suitable sign character and a finite order additive character), when evaluated at $s = 1$, is equal to the logarithm of an algebraic S-unit (the "Stark number"), up to some explicit multiplicative factor. It is also conjectured that this S-unit lies in a prescribed abelian extension of K. A direct computation of the Dirichlet series at $s = 1$, to a high accuracy, is difficult since the sum converges very slowly. In 1978, Shintani proposed an alternative way for computing good approximations of these Stark numbers by using an asymptotic expansion of the Barnes double Gamma function; and provided some interesting examples which supported the conjecture. In this talk we shall present another approach, due to Pierre Colmez, for computing good approximations of these Stark numbers in terms of an infinite sum of fast decreasing Fourier series coefficients of certain lattice Eisenstein series. This is joint work with Carlos Caralps Rueda who wrote a SAGE program that implements this Eisenstein series approach when the base field is real quadratic.

Garen Chiloyan

A classification of 2-adic Galois images of isogeny-torsion graphs

Let $\mathcal{E}$ be a $\mathbb{Q}$-isogeny class of elliptic curves defined over $\mathbb{Q}$. The isogeny graph associated to $\mathcal{E}$ is a graph which has a vertex for each elliptic curve in $\mathcal{E}$ and an edge for each $\mathbb{Q}$-isogeny of prime degree that maps one elliptic curve in $\mathcal{E}$ to another elliptic curve in $\mathcal{E}$, with the degree recorded as a label of the edge. An isogeny-torsion graph is an isogeny graph where, in addition, we label each vertex with the abstract group structure of the torsion subgroup over $\mathbb{Q}$ of the corresponding elliptic curve. In this talk, we will go over the classification of the 2-adic Galois image that occurs at each vertex of all isogeny-torsion graphs of elliptic curves defined over $\mathbb{Q}$

Michael Chou, Providence College

Rank growth in dihedral extensions

Selmer groups are important tools for studying the ranks of elliptic curves. We present a formulation of Selmer groups as a set of number fields cut out by certain local conditions. We then utilize field counting techniques allow us to prove positive density theorems for rank growth in quadratic twist families of elliptic curves with a 3-isogeny.

Vittoria Cristante, Tufts University (student)

An Improved Lower Bound for $GL_2(\mathbb{F}_\ell)$ Number Fields

A conjecture of Malle predicts an asymptotic for the number of fields with discriminant bounded by X with a given Galois group, G. In this talk, we consider the case when G is the general linear group of dimension 2 over a finite field, $GL_2(\mathbb{F}_\ell)$. It is known that number fields with this Galois group can arise from the $\ell-$torsion points of rational elliptic curves. Here we give an improved lower bound on the number of these fields.

John Cullinan, Bard College

The probability of non-isomorphic group structures of isogenous elliptic curves in finite field extensions

Let ℓ be a prime number and let E and E' be ℓ-isogenous elliptic curves defined over a finite field k of characteristic $p \neq \ell$. Suppose the groups $E(k)$ and $E'(k)$ are isomorphic, but $E(K) \not\simeq E'(K)$, where K is an ℓ-power extension of k. In a previous work we have shown that, under mild rationality hypotheses, the case of interest is when $\ell = 2$ and K is the unique quadratic extension of k.

In this talk we study the likelihood of such an occurrence from two points of view. First, we fix a pair of 2-isogenous elliptic curves E, E' over $\mathbf{Q}$ and study the proportion of primes p for which $E(\mathbf{F}_p) \simeq E'(\mathbf{F}_p)$ and $E(\mathbf{F}_{p^2}) \not\simeq E'(\mathbf{F}_{p^2})$. Second, we fix a prime p and study the proportion of pairs (E, E') of 2-isogenous curves over $\mathbf{F}_p$ for which $E(\mathbf{F}_p) \simeq E'(\mathbf{F}_p)$ and $E(\mathbf{F}_{p^2}) \not\simeq E'(\mathbf{F}_{p^2})$.

Alessandro Fazzari, Université de Montréal
Joint moments of the Riemann zeta function and its logarithmic derivative
We will discuss classical statistics for the Riemann zeta function when the averages are tilted by powers of zeta on the critical line. In particular, we will focus on the weighted statistics for the non-trivial zeros of the Riemann zeta function, blending together the theory of moments and that of n-th level density. This weighted approach allows for a better understanding of the interplay between zeros and large values of zeta.

Claire Frechette, Boston College
Metaplectic Ice: Lattice Models for General Metaplectic Whittaker Functions
Local Whittaker functions for principal series representations of reductive groups play an integral role in number theory and representation theory, and many of their applications extend to the metaplectic case, where reductive groups are replaced by their metaplectic covering groups. We will examine metaplectic Whittaker functions for covers of GL_r through the lens of a solvable lattice model, or ice model: a construction from statistical mechanics that provides a surprising bridge between spaces of Whittaker functions and representations of quantum groups. This story has been well studied before for the case of one particularly nice cover of GL_r, which eliminates complications arising from the center of the group. In this talk, we will show that the same types of connections hold for any metaplectic cover of GL_r, as well as examine how different choices of covering group interact with the center of GL_r to change the story.

Mathilde Gerbelli-Gauthier, McGill
Fourier Interpolation and the Weil Representation
In 2017, Radchenko-Viazovska proved a remarkable interpolation result for even Schwartz functions on the real line: such a function is entirely determined by its values and those of its Fourier transform at square roots of integers. We give a new proof of this result, exploiting the fact that Schwartz functions are the underlying vector space of the Weil representation W. This allows us to deduce the interpolation result from the computation of the cohomology of a certain congruence subgroup of $SL_2(\mathbb{Z})$ with values in W. This is joint work with Akshay Venkatesh.

Aditya Ghosh, Columbia University (student)
Counting prime ideals of a given degree in number fields
The talk deals with the question of counting prime ideals of a given degree in number fields K using the Chebotarev Density Theorem (CDT). The asymptotic expression involves the Parker numbers of the Galois action. Applying known explicit versions of CDT directly, one gets error terms that depend on the Galois Closure of K and can potentially be very large

if $[K : \mathbb{Q}]$ is large. We obtain error terms which are much smaller – polynomial bounds involving only the field K itself. To achieve this we use results from the Representation Theory of the symmetric group and bounds on inverse Kostka numbers. We conclude with a computational application – computing Parker numbers for the Galois group of an irreducible polynomial.

Eyal Goren, McGill University

Foliations on Shimura varieties

In joint work with E. De Shalit (Hebrew U) we developed a theory of foliations on Shimura varieties in positive characteristic, offering as "case studies" the examples of Hilbert modular varieties and unitary Shimura varieties. Those reveal intimate connections with modular forms mod p, Shimura varieties with parahoric level structure, inseparable morphisms and deformation theory. I will provide an overview, using two particular examples: Hilbert-Blumenthal surfaces and Picard modular surfaces.

Jon Grantham, IDA/CCS

No new Goormaghtigh primes up to 10^{500}

The Goormaghtigh conjecture states that the only two numbers which have non-trivial representations as repunits to two different bases are 31 and 8191. We show that no other primes less than 10^{500} satisfy that condition.

Asimina Hamakiotes, University of Connecticut (student)

Elliptic curves with complex multiplication and abelian division fields

Let K be an imaginary quadratic field, and let $\mathcal{O}_{K,f}$ be an order in K of conductor $f \geq 1$. Let E be an elliptic curve with CM by $\mathcal{O}_{K,f}$, such that E is defined by a model over $\mathbb{Q}(j_{K,f})$, where $j_{K,f} = j(E)$. In this talk, we classify the values of $N \geq 2$ and the elliptic curves E such that the division field $\mathbb{Q}(j_{K,f}, E[N])$ is an abelian extension of $\mathbb{Q}(j_{K,f})$.

Devadatta Ganesh Hegde, University of Minnesota (student)

Poles of Degenerate Eisenstein Series and the Spectral Decomposition of Automorphic Forms on GL(n)

We will give a short proof determining the Poles of degenerate Eisenstein Series. The residue at all but one of these poles is not square-integrable. We explain how these residues are canceled during the contour deformation of Langlands.

David Helm, Imperial College London
Finiteness for Hecke algebras of p-adic reductive groups
Let G be a p-adic group, and K a compact open subgroup of G. The Hecke algebra H(G,K) is the convolution algebra of (left,right) K-invariant functions on G. A famous result of Bernstein states that Hecke algebras of complex valued functions are Noetherian rings; a long-standing open question has been whether this remains true for Hecke algebras of functions taking values in more complicated rings such as $\mathbb{Z}_\ell$ or $\mathbb{Z}[1/p]$. In joint work with Jean-Francois Dat, Rob Kurinczuk, and Gil Moss, we explain how recent spectacular results of Fargues and Scholze allow us to give a positive answer to this question.

Anton Hilado, University of Vermont (student)
Non-regular loci in the Emerton-Gee stack for GL_2
Let K be a finite extension of the p-adic numbers. The reduced Emerton-Gee stack for GL_2 is a moduli stack whose F_p-points parametrize 2-dimensional mod p representations of the absolute Galois group of K. The irreducible components of this reduced Emerton-Gee stack are in correspondence with Serre weights, and work of Caraiani-Emerton-Gee-Savitt has established that the locus of mod p Galois representations with crystalline lifts of a fixed regular Hodge type is precisely one of these irreducible components (except when the Serre weight is Steinberg). In our work, for unramified K, we investigate the non-regular loci, which has positive codimension in the reduced Emerton-Gee stack. In particular we develop an algorithm which determines inclusions of these non-regular loci into the irreducible components of the reduced Emerton-Gee stack. This is joint work with Bellovin, Borade, Kansal, Lee, Levin, Savitt, and Wiersema.

Erman Isik, University of Ottawa
Modular approach to the Diophantine equation $x^p + y^p = z^3$ over some number fields
Solving Diophantine equations, in particular, Fermat-type equations is one of the oldest and most widely studied topics in mathematics. After Wiles' proof of Fermat's Last Theorem using his celebrated modularity theorem, several mathematicians have attempted to extend this approach to various Diophantine equations and number fields over several number fields. The method used in the proof of this theorem is now called "modular approach", which makes use of the relation between modular forms and elliptic curves. I will first briefly mention the main steps of the modular approach, and then report our asymptotic result (joint work with Özman and Kara) on the solutions of the Fermat-type equation $x^p + y^p = z^3$ over various number fields.

Paul Kinlaw, Dickinson College/UMaine Fort Kent

Sums Related to the Fibonacci Sequence

We investigate sums associated with the Fibonacci sequence F_n and the golden ratio ϕ. In particular, we study the sums $G(k) = \sum_{n=1}^{\infty} n^k/F_n$ and $H(k) = \sqrt{5} \cdot \mathrm{Li}_{-k}(1/\phi) = \sum_{n=1}^{\infty} n^k\sqrt{5}/\phi^n$. These sums generalize the reciprocal Fibonacci constant $\psi = G(0)$. We prove the asymptotic equivalence $G(k) \sim H(k)$, and moreover, $G(k)/H(k) = 1 + 1/5^{k+1} + O((\log \phi/\pi)^{k+1})$ as $k \to \infty$. We express $G(k) - H(k)$ as an alternating series, allowing us to compute values of these sums to high precision, and to prove that $G(k) > H(k)$ if and only if $k \geq 2$. We also generalize the results to their Lucas sequence analogues. As a tool, we establish a widely applicable explicit bound for polylogarithms of negative integer order. We find explicit bounds for the integer sequences $\{A_k\}_{k=1}^{\infty}$ and $\{B_k\}_{k=1}^{\infty}$ defined by $H(k)/\sqrt{5} = \mathrm{Li}_{-k}(1/\phi) = A_k + B_k\phi$. We also prove several results concerning the multiplicative structure of A_k and B_k. We show that $\{A_k \pmod m\}$ and $\{B_k \pmod m\}$ are periodic for every natural number m, and that the period is a divisor of $\lambda(m)$, where λ denotes the Carmichael function. This is joint work with Michael Morris and Samanthak Thiagarajan.

Matilde Lalin, Universite de Montreal

The distribution of values of cubic L-functions at $s = 1$

We investigate the distribution of values of cubic Dirichlet L-functions at $s = 1$. Following ideas of Granville and Soundararajan, and Dahl and Lamzouri for quadratic L-functions, we model values of $L(1, \chi)$ with the distribution of random Euler products $L(1, \mathbb{X})$ for certain family of random variables $\mathbb{X}(p)$ attached to each prime. We obtain a description of the proportion of $|L(1, \chi)|$ that are larger or that are smaller than a given bound, and yield more light into the Littlewood bounds. Unlike the quadratic case, there is a clear asymmetry between lower and upper bounds for the cubic case. This is joint work with Pranendu Darbar, Chantal David, and Allysa Lumley.

Antonio Lei, University of Ottawa

On ordinary isogeny graphs with level structure

Let l and p be two distinct prime numbers. The focus of this talk is a joint work with Katharina Mueller on l-isogeny graphs of ordinary elliptic curves defined over a finite field of characteristic p, together with a level structure. I will first discuss how as the level varies over all p-powers, the graphs form an Iwasawa-theoretic abelian p-tower. Secondly, I will discuss the structure of the “crater” of these graphs, generalizing previous results on volcano graphs. If time permits, I will discuss an inverse problem of graphs arising from the crater of l-isogeny graphs with level structures.

Robert Lemke Oliver, Tufts University
Uniform exponent bounds on the number of primitive extensions of number fields
A folklore conjecture asserts the existence of a constant $c_n > 0$ such that $N_n(X) \sim c_n X$ as $X \to \infty$, where $N_n(X)$ is the number of degree n extensions $K/\mathbb{Q}$ with discriminant bounded by X. This conjecture is known if $n \leq 5$, but even the weaker conjecture that there exists an absolute constant $C \geq 1$ such that $N_n(X) \ll_n X^C$ remains unknown and apparently out of reach.
Here, we make progress on this weaker conjecture (which we term the "uniform exponent conjecture") in two ways. First, we reduce the general problem to that of studying relative extensions of number fields whose Galois group is an almost simple group in its smallest degree permutation representation. Second, for almost all such groups, we prove the strongest known upper bound on the number of such extensions. These bounds have the effect of resolving the uniform exponent conjecture for solvable groups, sporadic groups, exceptional groups, and classical groups of bounded rank. This is forthcoming work that grew out of conversations with M. Bhargava.

Adam Logan, Government of Canada and ICERM
Surfaces with maximal Picard number
Work of Elkies and Schütt shows that Hecke eigenforms of weight 3 with integer coefficients are associated to supersingular K3 surfaces, that is, K3 surfaces whose Picard number is 20, the largest value allowed in characteristic zero. More generally, surfaces for which the rank of the Néron-Severi group is equal to the dimension of $\mathrm{H}^{1,1}$ are said to have *maximal Picard rank*. These are expected to correspond to certain modular forms of weight 3, in general with nonintegral coefficients. We discuss some constructions of such surfaces and their apparent modularity properties. This is joint work in progress with Asher Auel and John Voight.

David Lowry-Duda, ICERM & Brown University
Modular Murmurations
Recently, certain routine biases in averages of coefficients of L-functions have been observed from elliptic curves, Dirichlet characters, and others. These biases are called "Murmurations", and in this talk we discuss ongoing investigations into murmurations in new families of modular forms.

Jonathan Love, McGill
On isospectral quaternion orders
Schiemann proved in 1997 that a 3-dimensional integral lattice is determined up to isometry by the number of elements of each norm. However, in all higher dimensions, there exist many pairs of non-isometric lattices that are *isospectral*, meaning they have the same number of elements of norm n for all integers n (equivalently, they have the same theta function).

Given a quaternion algebra B_p over $\mathbb{Q}$ ramified at a single finite prime p, we show that if two maximal orders of B_p are isospectral, then they are isomorphic. This is joint work with Eyal Goren.

A. Anas Chentouf, Massachusetts Institute of Technology (student)
Jack Miller, Yale University (student)

Patterns of primes in joint Sato–Tate distributions.

For $j = 1, 2$, let $f_j(z) = \sum_{n=1}^{\infty} a_j(n)e^{2\pi inz}$ be a holomorphic, non-CM cuspidal newform of even weight $k_j \geq 2$ with trivial nebentypus. For each prime p, let $\theta_j(p) \in [0, \pi]$ be the angle such that $a_j(p) = 2p^{(k-1)/2} \cos\theta_j(p)$. The now-proven Sato–Tate conjecture states that the angles $(\theta_j(p))$ equidistribute with respect to the measure $d\mu_{\mathrm{ST}} = \frac{2}{\pi}\sin^2\theta\, d\theta$. We show that, if f_1 is not a character twist of f_2, then for subintervals $I_1, I_2 \subset [0, \pi]$, there exist infinitely many bounded gaps between the primes p such that $\theta_1(p) \in I_1$ and $\theta_2(p) \in I_2$. We also prove a common generalization of the bounded gaps with the Green–Tao theorem.

Eric Moss, Boston College

Computing Bianchi-Maass Forms

Maass cusp forms over Q are nonharmonic versions of classical modular cusp forms. They have been the subject of much numerical investigation, and the computational state of the art is Hejhal's algorithm. Bianchi-Maass cusp forms are defined over an imaginary quadratic field instead of Q. I have extended Hejhal's algorithm to compute weight 0 Bianchi-Maass forms over the class number 1 imaginary quadratic fields which allows us to compute the first example over a noneuclidean ring of integers.

Michael Mossinghoff, CCR Princeton

Ideal solutions in the Prouhet-Tarry-Escott problem

For given positive integers m and n with $m < n$, the *Prouhet–Tarry–Escott problem* asks if there exist two disjoint multisets of integers of size n having identical kth moments for $1 \leq k \leq m$; in the *ideal* case one requires $m = n - 1$, which is maximal. We describe some new searches for ideal solutions to the Prouhet–Tarry–Escott problem, especially solutions possessing a particular symmetry, both over $\mathbb{Z}$ and over the ring of integers of several imaginary quadratic number fields. This is joint work with D. Coppersmith, D. Scheinerman, and J. VanderKam.

Siva Nair, Universite de Montreal (student)
The Mahler measure of a family of polynomials
The Mahler measure of a Laurent polynomial $P(x_1, \ldots, x_n)$ is the integral of $\log |P|$ over the unit n-torus defined by $|x_i| = 1$ for all i. Interest in this quantity arose from the fact that the Mahler measures of certain polynomials are quite remarkable and come up as special values of L-functions. Computing the exact Mahler measure of a multivariate polynomial is usually very difficult (even for a computer!). However, it often boils down to evaluating certain polylogarithms, which in turn can be related to values of L-functions at integer points. In this talk, we will discuss recent results that express the Mahler measure of a family of polynomials with arbitrarily many variables in terms of a combination of polylogarithms evaluated on sixth roots of unity.

Zoe Batterman, Pomona College (student)
Aditya Jambhale, University of Cambridge (student)
Akash Narayanan, University of Michigan (student)
Christopher Yao, Yale University (student)
Modeling the vanishing of L-functions at the central point
The Katz-Sarnak philosophy states that statistics of zeros of L-function families near the central point as the conductors tend to infinity agree with those of eigenvalues of random matrix ensembles as the matrix size tends to infinity. For finite conductors, very different behavior can occur, as observed by S. J. Miller for zeros near the central point in elliptic curve families. This led to the excised model of Dueñez, Huynh, Keating, Miller, and Snaith, which accurately fits the data for quadratic twists of elliptic curves. This model accounts for the discretization of values of elliptic curve L-function at the central point by excising matrices whose characteristic polynomial at 1 is below a corresponding threshold (it also adjusts the size of the matrices for comparisons with finite levels by equating the lower order terms in statistics such as the one-level density and pair-correlation). We extend this model to families of quadratic twists of holomorphic cuspidal newforms of odd prime level, seeing the impact the weight, which controls the discretization of the value at the central point, has on the behavior of nearby zeros.

Tung Nguyen, Western University
On the arithmetic of Fekete polynomials of principal Dirichlet characters
Fekete polynomials have a rich history in mathematics. They first appeared in the work of Michael Fekete in his investigation of Siegel zeros of Dirichlet L-functions. In a previous study, we explored the arithmetic of generalized Fekete polynomials associated with primitive quadratic Dirichlet characters. We found that these polynomials possess a variety of interesting and important arithmetic and Galois-theoretic properties.
In this talk, we will introduce a different incarnation of Fekete polynomials, namely those associated with principal Dirichlet characters. Through numerical experiments, we examine their cyclotomic and non-cyclotomic factors and identify some of their roots in the unit

circle. We also investigate their modular properties and special values. Last but not least, based on both theoretical and numerical data, we propose a precise question on the structure of the Galois group of these Fekete polynomials. This is based on joint work with Shiva Chidambaram, Jan Minac, and Nguyen Duy Tan.

Peikai Qi, Michigan State University (student)
Iwasawa lambda Invariant and Massey product
How does the class group of the number field change in field extensions? This question is too wild to have a uniform answer, but there are some situations where partial answers are known. I will compare two such situations. First, in Iwasawa theory, instead of considering a single field extension, one considers a tower of fields and estimates the size of the class groups in the tower in terms of some invariants called λ and μ. Second, in a paper of Lam-Liu-Sharifi-Wake-Wang, they relate the relative size of Iwasawa modules to values of a "generalized Bockstein map", and further relate these values to Massey products in Galois cohomology in some situations. I will compare these two approaches to give a description of the cyclotomic Iwasawa λ-invariant of some imaginary quadratic fields and other fields in terms of Massey products.

Sarah Peluse, University of Michigan
Arithmetic patterns in dense sets
Some of the most important problems in combinatorial number theory ask for the size of the largest subset of the integers in an interval lacking points in a fixed arithmetically defined pattern. One example of such a problem is to prove the best possible bounds in Szemerédi's theorem on arithmetic progressions, i.e., to determine the size of the largest subset of $\{1, ..., N\}$ with no nontrivial k-term arithmetic progression $x, x+y, ..., x+(k-1)y$. Gowers initiated the study of higher order Fourier analysis while seeking to answer this question, and used it to give the first reasonable upper bounds for arbitrary k. In this talk, I'll discuss recent progress on quantitative polynomial, multidimensional, and nonabelian variants of Szemerédi's theorem and on related problems in number theory, harmonic analysis, and ergodic theory.

Tristan Phillips, Dartmouth College
Average analytic ranks of elliptic curves over number fields
In this talk I will discuss a bound on the average analytic rank of isomorphisms classes of elliptic curves over an arbitrary number field. Some key features of the argument that will be highlighted in the talk are counting points of weighted projective stacks and estimating sums of Hurwitz-Kronecker class numbers.

Firdavs Rakhmonov, University of Rochester (student)
Distribution of similar configurations in subsets of F_q^d
Let $\mathbb{F}_q$ be a finite field of order q and E be a set in $\mathbb{F}_q^d$. The distance set of E is defined by $\Delta(E) := \{\|x-y\| : x, y \in E\}$, where $\|\alpha\| = \alpha_1^2 + \cdots + \alpha_d^2$. Iosevich, Koh and Parshall (2018) proved that if $d \geq 2$ is even and $|E| \geq 9q^{d/2}$, then

$$\mathbb{F}_q = \frac{\Delta(E)}{\Delta(E)} = \left\{ \frac{a}{b} : a \in \Delta(E),\ b \in \Delta(E) \setminus \{0\} \right\}.$$

In other words, for each $r \in \mathbb{F}_q^*$ there exist $(x, y) \in E^2$ and $(x', y') \in E^2$ such that $\|x-y\| \neq 0$ and $\|x' - y'\| = r\|x - y\|$.
Geometrically, this means that if the size of E is large, then for any given $r \in \mathbb{F}_q^*$ we can find a pair of edges in the complete graph $K_{|E|}$ with vertex set E such that one of them is dilated by $r \in \mathbb{F}_q^*$ with respect to the other. A natural question arises whether it is possible to generalize this result to arbitrary subgraphs of $K_{|E|}$ with vertex set E and this is the goal of this paper.
In this talk, I'll explain the proof of the problem for k-paths ($k \geq 2$), simplexes and 4-cycles. We are using a mix of tools from different areas such as enumerative combinatorics, group actions and Turán type theorems.

Martí Roset Julià, McGill (student)
Dihedral long root local A-packets of G_2 via theta correspondence.
Let G be a split exceptional group of type G_2. Arthur's Conjecture describes the constituents of the square integrable automorphic representations of G. It decomposes this space as a direct sum of subspaces consisting of near equivalence classes of representations. These subspaces, called A-packets, are indexed by certain morphisms called A-parameters.
We will focus on a particular type of A-parameters of G, the dihedral long root A-parameters. We will explain that they factor through A-parameters for the group $\mathrm{PU}_3 \rtimes \mathbb{Z}/2\mathbb{Z}$. Motivated by this, we will use the exceptional theta correspondence between $\mathrm{PU}_3 \rtimes \mathbb{Z}/2\mathbb{Z}$ and G_2 to propose a construction of the local representations of G that appear in the corresponding A-packets. This is joint work with Raúl Alonso, Qiao He, and Mishty Ray and is part of a larger project (involving other authors) that aims to prove Arthur's Conjecture for this type of A-parameters.

Giovanni Rosso, Concordia Univeristy
Overconvergent Eichler–Shimura morphism for families of Siegel modular forms
Classical results of Eichler and Shimura decompose the cohomology of certain local systems on the modular curve in terms of holomorphic and anti-holomorphic modular forms. A similar result has been proved by Faltings' for the étale cohomology of the modular curve and Falting's result has been partly generalised to Coleman families by Andreatta–Iovita–Stevens. In this talk, based on joint work with Hansheng Diao and Ju-Feng Wu, I will explain

how one constructs a morphism from the overconvergent cohomology of GSp_{2g} to the space of families of Siegel modular forms. This can be seen as a first step in an Eichler–Shimura decomposition for overconvergent cohomology and involves a new definition of the sheaf of overconvergent Siegel modular forms using the Hodge–Tate map at infinite level.

Subham Roy, Université de Montréal (student)

Areal Mahler measure and some examples

The (logarithmic) Mahler measure of a non-zero rational polynomial P in n variables is defined as the mean of $\log|P|$ (with respect to the normalized arclength measure) restricted to the standard n-torus ($\mathbb{T}^n = \{(x_1, \ldots, x_n) \in (\mathbb{C}^*)^n : |x_i| = 1, \forall\ 1 \leq i \leq n\}$). It has been related to special values of L-functions. Pritsker (2008) defined a natural counterpart of the Mahler measure, which is obtained by replacing the normalized arclength measure on the standard n-torus by the normalized area measure on the product of n open unit disks. It inherits many nice properties, such as the multiplicative ones. In this talk, we will investigate some similarities and differences between the two. We will also discuss some evaluations of the areal Mahler measure of multivariable polynomials, which also yield special values of L-functions. This is an ongoing joint work with Prof. Matilde Lalín.

Ester Sgallová, Michigan State University (student)

Prime sequences in discretely ordered principal ideal subring of Q[x]

This talk aims to show the constructions of discretely ordered principal ideal sub-rings of Q[x] with various types of prime behavior. The first construction shows such subring whose primes will just be primes in the ring of integers. The second construction will give the subring with a cofinal set of prime pairs. Moreover, we will talk about the generalization of these results.

Arseniy Sheydvasser, Bates College

Logical Rigidity of Arithmetic Groups

Arithmetic groups (particularly higher-rank ones) are rigid in many different algebraic and geometric senses. (Consider results such as the Mostow and Margulis Rigidity Theorems.) In this talk, I will discuss a bit about how higher-rank arithmetic groups are rigid from the point of view of model theory, including some recent work due to Chen Meiri and myself.

Jae Hyung Sim, Boston University (student)
Dedekind-Rademacher Cocycle and Explicit Class Field Theory
Darmon and Vonk's theory of rigid cocycles have provided a p-adic analytic method for addressing questions over real quadratic fields analogous to the classical theory of CM. These methods have been shown to provide p-adic analytic cocycles whose special values correspond to Gross-Stark units as well as computationally coincide with Stark-Heegner points. In this talk, I will outline the construction of the Dedekind-Rademacher cohomology class from classical Kato-Siegel units and its relevance to explicit class field theory over real quadratic fields via the work of Darmon-Vonk-Pozzi and Dasgupta-Kakde.

Jakob Streipel, University of Maine
Zero-density estimate for Hecke–Maass L-functions
In this talk we will discuss recent work, joint with Sheng-Chi Liu, in which we establish an asymptotic formula for the second moment of the L-functions attached to Hecke–Maass cusp forms, twisted by a Hecke eigenvalue. In particular we will talk about how to use this moment, together with Selberg's method of counting zeros, to establish a zero-density estimate for these L-functions in the spectral aspect. This kind of unconditional result on the distribution of the zeros of L-functions is occasionally enough to dispense with assumptions of Riemann Hypotheses in certain applications.

Arvind Suresh, University of Arizona
Genus g curves with rank 8g over real cyclotomic fields
We present a refinement of a well known construction of Mestre and Shioda, which yields families of genus g curves having rank at least 8g over certain real cyclotomic fields. This improves, for g sufficiently large, on the previous rank record of 4g+7 for infinite families of curves, due to Shioda.

Shiang Tang, Dartmouth College
Lifting G-valued Galois representations when $\ell \neq p$
We study the universal lifting spaces of local Galois representations valued in arbitrary reductive group schemes when $\ell \neq p$. In particular, under certain technical conditions applicable to any root datum we construct a canonical smooth component in such spaces, generalizing the minimally ramified deformation condition previously studied for classical groups. Our methods involve extending the notion of isotypic decomposition for a GL_n-valued representation to general reductive group schemes. To deal with certain scheme-theoretic issues coming from this notion, we are led to a detailed study of certain families of disconnected reductive groups, which we call *weakly reductive* group schemes. Our work can be used to produce geometric lifts for global Galois representations, and we illustrate this for G_2-valued representations.

Eric Urban, Columbia

On Euler systems for p-ordinary Galois representations

In past works, I gave a new construction of the cyclotomic Euler system using Eisenstein congruences and the p-adic Langlands correspondance for $GL(2, \mathbb{Q}_p)$. The goal of this lecture is to outline the strategy to generalize this method to construct new Euler systems in the ordinary setting and relate them to the corresponding p-adic L-function. I will also describe some of the main technical ingredients which are needed for this construction.

Zoe Batterman, Pomona College (student)
Aditya Jambhale, University of Cambridge (student)
Akash Narayanan, University of Michigan (student)
Christopher Yao, Yale University (student)

Biases in second moments of elliptic curves

Michel proved for non-CM one-parameter families of elliptic curves that the second moment of solutions modulo p is $p^2 + O(p^{3/2})$. Analyzing the lower-order terms in the second moment yields results for lower-order terms in the n-level densities of Katz and Sarnak, which describe the behavior of the zeros on the critical line of the associated L-functions. The *bias conjecture* predicts the largest lower-order term in the second moment expansion that does not average to 0 is on average negative. The conjecture has been verified for many families, but always ones with computable Legendre sums and thus not "generic."
We found a family, $\mathcal{E}_t : y^2 = x^3 + x + t^3$, whose second moment is $p^2 + p$ for primes equivalent to 2 modulo 3; this deviates from the bias conjecture. The lower order terms at the primes 1 modulo 3 appear to be negative enough so that a modified bias conjecture, where we average over the primes, is true. The analysis requires numerous results on Legendre and Gauss sums, as well as converting these curves to nonic surfaces and using powerful methods from algebraic geometry to count the number of points, dealing with issues arising from singular fibers at infinity.

Benjamin York, University of Connecticut (student)

On the adelic image of Galois representations attached to elliptic curves with complex multiplication

Let E be an elliptic curve defined over a number field K, and let ρ_E be the adelic Galois representation attached to E/K. The goal of the so-called Mazur's Program B is to classify the possibilities for the image of ρ_E as a subgroup of $GL(2, \widehat{\mathbb{Z}})$, up to conjugation. In this talk, we will discuss a method for computing adelic images of elliptic curves over $\mathbb{Q}$ with complex multiplication. This is joint work with Alvaro Lozano-Robledo.

Robin Zhang, Massachusetts Institute of Technology

Harris-Venkatesh plus Stark

The class number formula describes the behavior of the Dedekind zeta function at $s = 0$ and $s = 1$. The Stark conjecture extends the class number formula, describing the behavior of Artin L-functions and p-adic L-functions at $s = 0$ and $s = 1$ in terms of units. The Harris–Venkatesh conjecture describes the residue of Stark units modulo p, giving a modular analogue to the Stark and Gross conjectures while also serving as the first verifiable part of the broader conjectures of Venkatesh, Prasanna, and Galatius. In this talk, I will draw an introductory picture of a unified conjecture combining Harris–Venkatesh and Stark, which is now resolved for imaginary dihedral modular forms.